Mahindra University Hyderabad

École Centrale School of Engineering
Minor-II exam

Program: B. Tech. **Branch: CM, NT, ECM, ECE** **Year: IV** **Semester: I**
Subject: Computational Methods for Fluid Flow (MA4110)

Date: 30/10/2026 **Start Time: 2:00 PM**

Time Duration: 1.5 Hours **Max. Marks: 20**

Instructions:

1) Answer all the questions.

2) All questions are self-explanatory; no clarification will be provided during the exam.

3) Use of one non-programmable scientific calculator is allowed.

4) Notation: $\Delta_{i+1/2} = \bar{u}_{i+1} - \bar{u}_i$, $\nu = a\Delta t/\Delta x$, and Burgers' equation is $u_t + (u^2/2)_x = 0$.

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1. (6 points) Consider Burgers' equation with the Riemann data

$$u(x,0) = \begin{cases} u_L = 1, & x \le 0, \\ u_R = 0, & x > 0, \end{cases}$$

and Neumann (zero gradient) boundary conditions, on a uniform grid $x_j = j\Delta x$. Let $\lambda = \Delta t/\Delta x = 0.5$.

   (a) (3 points) The quasi-linear form $u_t + uu_x = 0$ discretised with first-order upwinding gives

$$u_j^{n+1} = u_j^n - \lambda\, u_j^n \left(u_j^n - u_{j-1}^n\right).$$

   Show that $u_j^n = u_j^0$ for all n and j. What is the numerical shock speed, and what is the exact shock speed?

   (b) (2 points) Using the upwind scheme $u_j^{n+1} = u_j^n - \lambda\left(F_{j+1/2} - F_{j-1/2}\right)$ with $F_{j+1/2} = \frac{1}{2}(u_j^n)^2$, compute u_1^1. Explain in one line why this scheme moves the shock while the scheme in (a) does not.

   (c) (1 point) State the Lax-Wendroff theorem.

2. (5 points) For a convex flux f, the Godunov and Rusanov fluxes at an interface with states u_L, u_R are

$$F^G = \begin{cases} \min\limits_{u_L \le u \le u_R} f(u), & u_L \le u_R, \\ \max\limits_{u_R \le u \le u_L} f(u), & u_L > u_R, \end{cases} \qquad F^{\mathrm{Rus}} = \frac{1}{2}\big(f(u_L) + f(u_R)\big) - \frac{1}{2} s_{\max}(u_R - u_L),$$

with $s_{\max} = \max\left(|f'(u_L)|, |f'(u_R)|\right)$. Take $f(u) = u^2/2$ and $(u_L, u_R) = (-1, 2)$.

   (a) (3 points) Is the exact Riemann solution a shock or a rarefaction? Compute F^G.

   (b) (2 points) Compute F^{Rus}.

3. (5 points) Five cells carry the averages $\bar{u}_1 = 0,\ \bar{u}_2 = 1,\ \bar{u}_3 = 3,\ \bar{u}_4 = 4,\ \bar{u}_5 = 4$. The minmod slope is

$$\Delta x\, \sigma_i = \mathrm{minmod}\left(\Delta_{i-1/2}, \Delta_{i+1/2}\right).$$

   (a) (2 points) Compute $\Delta x\, \sigma_i$ for $i = 2, 3, 4$ and present your answers in a table.

   (b) (2 points) Compute the interface states $u_{5/2}^R$ and $u_{7/2}^L$ in cell 3, and check that they lie between the neighbouring averages.

   (c) (1 point) Compute the unlimited centred slope $\frac{1}{2}(\Delta_{i-1/2} + \Delta_{i+1/2})$ in cell 4 and the resulting $u_{9/2}^L$. What goes wrong, and how does the limiter prevent it?

P.T.O.

4. (4 points) The candidate third-order reconstructions of $u^L_{i+1/2}$ are

$$u^L_{i+1/2}: \quad \tfrac{1}{3}\bar{u}_{i-2} - \tfrac{7}{6}\bar{u}_{i-1} + \tfrac{11}{6}\bar{u}_i, \qquad -\tfrac{1}{6}\bar{u}_{i-1} + \tfrac{5}{6}\bar{u}_i + \tfrac{1}{3}\bar{u}_{i+1}, \qquad \tfrac{1}{3}\bar{u}_i + \tfrac{5}{6}\bar{u}_{i+1} - \tfrac{1}{6}\bar{u}_{i+2}.$$

Consider the cell averages $\bar{u}_{i-2} = 0,\ \bar{u}_{i-1} = 0,\ \bar{u}_i = 0,\ \bar{u}_{i+1} = 2,\ \bar{u}_{i+2} = 2,\ \bar{u}_{i+3} = 2.$

(a) (2 points) Using third-order ENO, find the stencil chosen for $u^L_{i+1/2}$ and its value.

(b) (2 points) Using third-order ENO, find the stencil chosen for $u^R_{i+1/2}$ and its value.

End of exam