Mahindra University Hyderabad

École Centrale School of Engineering

Minor-I exam

Program: B. Tech. **Branch: CM, NT, ECM, ECE** **Year: IV** **Semester: I**

Subject: Computational Methods for Fluid Flow (MA4110)

Date: 16/09/2026 **Start Time: 4:30 PM**

Time Duration: 1.5 Hours **Max. Marks: 20**

Instructions:

1) Answer all the questions.

2) All questions are self-explanatory; no clarification will be provided during the exam.

3) Use of one non-programmable scientific calculator is allowed.

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1. (4 points) Classify each first-order PDE below into one of these four categories (Linear, Quasi-linear, Semi-linear, Non-linear) and justify your choice:
   (a) $u_t + \cos(u)\, u_x = 0$
   (b) $u_t + u^2\, u_x = 0$
   (c) $u_t + 10\, u_x = 0$
   (d) $u_t + \sqrt{1 + (u_x)^2} = 0$

2. (2 points) Classify the following second-order PDE into one of these three categories (Elliptic, Parabolic, Hyperbolic) and justify your choice:
   (a) $u_{tt} = c^2 u_{xx}, \quad c \neq 0$
   (b) $u_{xx} - 4\, u_{xy} + 3\, u_{yy} = 0$

3. (4 points) For the inviscid Burgers' equation
$$u_t + uu_x = 0, \qquad u(x,0) = u_0(x),$$
consider the initial profile
$$u_0(x) = -xe^{-x^2}.$$
Determine the breaking time t_s and the corresponding breaking coordinate x_s, where x_s is the location at which the characteristics first intersect.

4. (4 points) For the PDE
$$u_t + t\, u_x = 0, \qquad u(x,0) = u_0(x),$$
find the characteristic curves $x(t; x_0)$ and sketch the family of characteristics in the (x,t)-plane for
$$x_0 = -2, -1, 0, 1, 2.$$

5. (6 points) Answer any one of the following two questions:
   (a) (6 points) Consider the advection equation on the domain $x \in [0,1]$,
$$\frac{\partial u}{\partial t} + 2\,\frac{\partial u}{\partial x} = 0,$$
with initial condition
$$u(x,0) = \begin{cases} 1, & x \leq 0.4, \\ 0, & x > 0.4, \end{cases}$$
and inflow boundary condition $u(0,t) = 1$. Use $dx = 0.2$ and $dt = 0.05$. Using the first-order upwind scheme, compute the numerical solution after two time steps and present the nodal values in a table. Also state the CFL number.

P.T.O.

(b) (6 points) Consider the implicit upwind scheme for the advection equation $u_t + au_x = 0$ with $a > 0$:

$$\frac{u_j^{n+1} - u_j^n}{\Delta t} + a\frac{u_j^{n+1} - u_{j-1}^{n+1}}{\Delta x} = 0.$$

Using the Fourier mode $u_j^n = \hat{u}^n e^{\mathrm{i}j\xi}$, derive the amplification factor $G(\xi)$ and show that

$$|G(\xi)|^2 = \frac{1}{(1 + \nu - \nu\cos\xi)^2 + (\nu\sin\xi)^2}, \qquad \nu = \frac{a\Delta t}{\Delta x}.$$

Hence, prove that $|G(\xi)| \leq 1$ for all $\xi \in [-\pi, \pi]$ and determine whether the scheme imposes any restriction on Δt.

End of exam