Foundational Mathematics for ML (MA2221)

Assignment (Introduction to Plotting: Matplotlib Basics)

Instructor: **Biswarup Biswas**

Instructions:

- All problems must be solved using **NumPy** and **Matplotlib**.
- Use **functions, loops, and conditional statements** where specified.
- Label all axes, add titles, and include legends wherever appropriate.
- Problems are marked with difficulty levels: Easy ([E]), Moderate ([M]), Challenging ([C]).

1. [E] Using `numpy` and `matplotlib`:
   (a) Generate $x =$ `np.linspace`$(0,\ 2\pi,\ 200)$ and plot $y = \sin(x)$.
   (b) Add a title *"Plot of sin(x)"*, label the axes x and $\sin(x)$, and display a grid.
   (c) On the **same figure**, also plot $y = \cos(x)$ with a dashed red line. Add a legend identifying both curves.

2. [E] Customise the appearance of a line plot:
   (a) Plot $y = \sin(x)$ over $[0, 2\pi]$ with a **green** colour, linewidth 2, and circle markers of size 4.
   (b) Save the resulting figure to a file named `sine_custom.png` using `plt.savefig`.
   (c) Change the figure size to $(8, 3)$ inches. Comment on how the aspect ratio affects the perceived frequency of the wave.

3. [M] Scatter plots and shading:
   (a) Generate $x =$ `np.linspace`$(0, 10, 30)$ and noisy data $y = x + \varepsilon$ where $\varepsilon \sim \mathcal{N}(0, 1)$. Produce a scatter plot.
   (b) On a separate figure, plot $\sin(x)$ and $\cos(x)$ over $[0, 2\pi]$ and shade the region between them using `fill_between` with `alpha=0.3`.
   (c) Extend part (b): also plot an *exact* curve $\sin(x)$ alongside a *numerical* approximation $\sin(x) + 0.1\sin(5x)$. Shade the error region and label both curves.

4. [E] Discrete and statistical visualisations:
   (a) Create a bar chart for the categories $\{A, B, C, D\}$ with values $\{10, 15, 7, 12\}$. Add appropriate axis labels.
   (b) Generate 1000 samples from a standard normal distribution and display a histogram with 20 bins.
   (c) On a 1×2 subplot grid, place the bar chart in the left panel and the histogram in the right panel. Use `tight_layout`.

5. [M] Subplots and mathematical typesetting:

(a) Create a 1×2 subplot figure. In the left panel plot $\sin(x)$; in the right panel plot $\cos(x)$, each over $[0, 2\pi]$. Title each panel appropriately.

(b) Plot the Gaussian curve

$$f(x) = e^{-x^2}, \quad x \in [-5, 5].$$

Use LaTeX strings for the x-axis label (\$$x$\$), y-axis label (\$$e^{-x^2}$\$), and title.

(c) Extend part (b) to a 1×2 subplot: left panel shows $f(x) = e^{-x^2}$; right panel shows $g(x) = xe^{-x^2}$. Use LaTeX labels throughout.

6. [M] Logarithmic plots for convergence analysis:

(a) Given mesh sizes $h = [0.1,\ 0.05,\ 0.025,\ 0.0125]$ and errors $e = h^2$, produce a **log–log** plot. Add grid lines on both axes (`which="both"`).

(b) Compute the numerical convergence order using

$$p = \frac{\log(e_i/e_{i+1})}{\log(h_i/h_{i+1})}.$$

Use a `for` loop and print each estimated order.

(c) Given $n = [10, 20, 40, 80, 160]$ and error $e = e^{-0.3n}$, produce a **semi-log** (y-axis logarithmic) plot with markers. Label all axes.

7. [M] Two-dimensional visualisations:

(a) Over the domain $[-2, 2] \times [-2, 2]$ with 100 points per axis, compute

$$Z = e^{-(X^2+Y^2)}.$$

Produce a **contour** plot with 20 levels and add a colour bar.

(b) Repeat part (a) using `contourf` (filled contour). Comment on the visual difference.

(c) Generate a random 20×20 matrix A and display it as a **heatmap** using `imshow` with `origin="lower"` and a colour bar.

8. [C] Three-dimensional surface plots:

(a) Using the Gaussian $Z = e^{-(X^2+Y^2)}$ from Question 7, create a 3-D surface plot with the `viridis` colourmap. Add a colour bar.

(b) On $[0, 1] \times [0, 1]$, define

$$Z = \sin(\pi X)\sin(\pi Y).$$

Plot the surface using the `plasma` colourmap. Interpret the shape: what PDE eigenfunction does this resemble?

(c) Create a **2-panel figure**: left panel shows the filled contour (`contourf`) of Z from part (b); right panel shows the 3-D surface. Use an `if` statement to print `"Positive region"` if the maximum of Z exceeds 0.9, and `"Check values"` otherwise.

::: End of Assignment Sheet :::